

Maximum Likelihood vs Threshold Frame Synchronization

B. K. Levitt

Communications Systems Research Section

Mariner 10 (MVM'73) uses a threshold-type algorithm to acquire frame synchronization. In reviewing this scheme, the question arose as to why a threshold approach was selected over a maximum likelihood detection scheme. This paper answers that question in the context of an uncoded telemetry link over a binary symmetric channel. For a given communication link it is demonstrated that, at the expense of a variable acquisition time, a threshold scheme can simultaneously achieve a lower probability of false sync acquisition and a smaller expected acquisition time than its maximum likelihood counterpart.

I. The Problem

In a binary signaling scheme, frame synchronization is usually provided by prefixing each frame of transmitted data (M bits) with a fixed binary pattern or sync word (L bits). At the receiver, frame synchronization is acquired by locating the noisy replicas of the transmitted sync words periodically imbedded in the received data stream.

Suppose we are dealing with uncoded data received over a binary symmetric channel (crossover probability ϵ): the optimum frame sync decision is then based on the Hamming distance metric. In particular, we examine successive binary L -tuples $\mathbf{p}_1, \mathbf{p}_2, \dots$ within the received bit stream (see Fig. 1): we know that one of the first M such L -tuples must be received sync word \mathbf{p}_{m^*} . Statistically, m^* is uniformly distributed over $(1, M)$. To compare these received L -tuples with the sync word pattern \mathbf{s} , we form the likelihood parameters D_1, D_2, \dots , where D_m is the Hamming distance between \mathbf{p}_m and \mathbf{s} . Typically, D_{m^*}

will be near 0, while the other $(M-1)$ parameters D_m within a frame of received data will be near $L/2$ (assume \mathbf{s} is a Barker (Ref. 1) or Neuman-Hofman (Ref. 2) sequence, and the $(M-L)$ information bits in each frame of transmitted data are independent, equally likely 1's and 0's). We acquire frame synchronization by correctly identifying m^* over $(1, M)$: this is a detection problem with many solutions, some of which are discussed below.

II. Some Solutions

Suppose we are constrained to make a sync decision over a *single* frame of received data. Then the optimum maximum likelihood (ML) approach is to decide $m^* = \hat{m}$ if

$$D_{\hat{m}} = \min \{ D_m; m = 1, 2, \dots, M \} \quad (1)$$

The advantage of such a scheme is that the sync acquisition time is deterministic: assuming a unique minimum exists, we are guaranteed a sync decision will be made

after observing one frame of received data. On the other hand, if $D_{\hat{m}}$ is not particularly close to zero, we are still forced to make a sync decision, even though our confidence in that decision is low. Thus, if the bit error probability ϵ is too large, the *ML* scheme may have an unacceptably large probability of false sync acquisition, $Pr[FS]$.

We can improve $Pr[FS]$ by making a joint *ML* sync decision over more than one frame of received data (say, N frames): thus we can decide $m^* = \hat{m}$ if

$$\sum_{k=0}^{N-1} D_{\hat{m}+kM} = \min \left\{ \sum_{k=0}^{N-1} D_{m+kM} : m = 1, 2, \dots, M \right\} \quad (2)$$

The averaging over several frames of received data implied by the summation in Eq. (2) provides a degree of noise immunity and results in a significant decrease in $Pr[FS]$. However, our sync acquisition time is now N frames of received data, and our storage requirements are increased proportionally.

An alternative approach is to delay making a sync decision until we are reasonably confident it is correct. To this end, we can successively compare D_1, D_2 , etc., with some predetermined threshold T : we decide that $\mathbf{p}_{\hat{m}}$ is a received sync word if $D_{\hat{m}}$ is the *first* metric to satisfy the threshold test

$$D_m \leq T \quad (3)$$

The selection of the threshold T involves a tradeoff between frame sync reliability and acquisition time. Clearly, $Pr[FS]$ decreases as T gets smaller. However, if T is too small, we may not recognize the first few received sync words due to excessive ($T + 1$ or more) bit errors within them: then the sync acquisition time (now a random variable) may be several frames of received data. Note that if the first received sync word \mathbf{p}_{m^*} is the first received L -tuple to satisfy the threshold test, the corresponding acquisition time is m^*/M frames: consequently, the threshold synchronization scheme above has a *minimum* expected acquisition time of

$$\frac{\overline{m^*}}{M} = \frac{M+1}{2M} \text{ frames} \quad (4)$$

(assuming a correct sync decision is made). The design criterion for a given telemetry link (fixed M, L, ϵ) is to select a threshold T for which $Pr[FS]$ is acceptably small and the mean acquisition time is of the order of $\frac{1}{2}$ frame.

As in the maximum likelihood approach, if storage capacity permits, a more reliable sync decision can be made by using a joint threshold test over N frames of received data: determine the first location \hat{m} which satisfies the test

$$\sum_{k=0}^{N-1} D_{m+kM} \leq T \quad (5)$$

III. Comparison of *ML* and Threshold Synchronization

The maximum likelihood synchronization technique above is optimum, in the sense that it minimizes the probability of false sync acquisition, *only when the observable is a fixed amount of received data*. A threshold scheme can outperform its *ML* counterpart operating on the same N -fold Hamming distance metric,

$$\sum_{k=0}^{N-1} D_{m+kM}$$

provided the threshold T is properly chosen, at the expense of having a variable acquisition time.

*The general hypothesis is that for any ϵ, M, L , and N , there exists a T such that the $Pr[FS]$ and the mean acquisition time will both be smaller than in the *ML* case.* I am not prepared to prove this general statement; however, it is shown to hold below in the particular case $N = 1$, $M = 7056$, and $L = 31$. (This combination of M and L corresponds to the Mariner 10 high-rate video telemetry mode, which was of interest during my initial investigation of the frame sync problem.)

Using union bounding techniques, one can derive upper bounds for the probability of false sync acquisition for the *ML* scheme of Eq. (1) and the threshold approach of Eq. (3) (derivations are similar to the appendix of Ref. 3):

$$Pr[FS] \Big|_{ML} < (M-1)2^{-L}(1-\epsilon)^L \sum_{k=0}^L \binom{L}{k} \left(\frac{\epsilon}{1-\epsilon} \right)^k \sum_{\ell=0}^k \binom{L}{\ell} \quad (6)$$

$$Pr[FS] \Big|_T < (M-1)2^{-(L+1)} \sum_{k=0}^T \binom{L}{k} + \frac{\min(A, B)}{1-A} \quad (7)$$

where

$$A \equiv 1 - (1 - \epsilon)^L \sum_{k=0}^T \binom{L}{k} \left(\frac{\epsilon}{1 - \epsilon} \right)^k \quad (8)$$

$$B \equiv (M - 1) 2^{-L} \sum_{k=0}^T \binom{L}{k} \quad (9)$$

For sufficiently small ϵ , these bounds should be fairly tight.

The acquisition time is fixed at 1 frame of received data in the *ML* scheme. In the threshold case, a simple Markov analysis (similar to Section II, Appendix, Ref. 3) yields the result

$$\text{mean acquisition time} \Big|_T = \left(\frac{M+1}{2M} + \frac{A}{1-A} \right) \text{frames} \quad (10)$$

provided $Pr[FS] \Big|_T \ll 1$.

The formulas in Eqs. 6–10 were used to generate Figs. 2 and 3. They show that for any value of ϵ , a value of T

can be found to make the threshold scheme superior to the *ML* approach:

$$Pr[FS] \Big|_T \leq Pr[FS] \Big|_{ML} \quad (11)$$

$$\text{mean acquisition time} \Big|_T \leq 1 \text{ frame} \quad (12)$$

For small bit error rates ($\epsilon < 10^{-3}$), the probabilities of false sync acquisition for the *ML* and $T = 0$ threshold schemes are comparable (actually, $Pr[FS] \Big|_{T=0} \sim \frac{1}{2}$

$Pr[FS] \Big|_{ML}$); however, there is a factor of 2 advantage

for the threshold approach with regard to the mean acquisition time. As ϵ increases from 10^{-3} to 10^{-2} , the mean acquisition time for the $T = 0$ case rises toward 1 frame; simultaneously, $Pr[FS] \Big|_{T=0}$ increases much

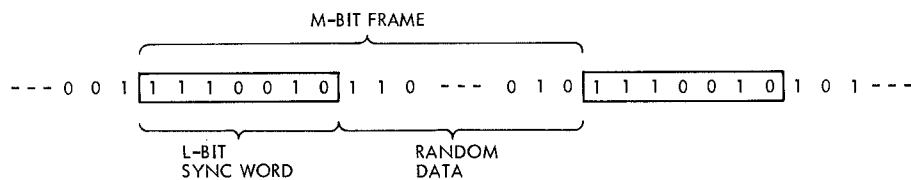
more slowly than $Pr[FS] \Big|_{ML}$, yielding more than an

order-of-magnitude performance advantage at $\epsilon = 10^{-2}$. In the region $9.6 \times 10^{-3} < \epsilon < 1.3 \times 10^{-2}$, the $T = 0$ and $T = 1$ thresholds both satisfy Eqs. (11) and (12); over the range $1.3 \times 10^{-2} < \epsilon < 2.5 \times 10^{-2}$, these constraints uniquely require the threshold $T = 1$. The graphs indicate where T must be increased as ϵ increases further. Of course, the bounds on $Pr[FS]$ become progressively looser as ϵ becomes large.

References

1. Barker, R. H., "Group Synchronization of Binary Digital Systems," *Communication Theory*, edited by Jackson, Butterworth, London, pp. 273-287, 1953.
2. Neuman, F., and Hofman, L., "New Pulse Sequences With Desirable Correlation Properties," *Proc. Nat. Telemetry Conf.*, Washington, D.C., pp. 277-282, Apr. 1971.
3. Levitt, B. K., "Frame Synchronization Performance Analysis for MVM'73 Uncoded Telemetry Modes," in *The Deep Space Network*, Technical Report 32-1526, Vol. XIX, pp. 126-136, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1974.

TRANSMITTED BIT STREAM:



RECEIVED BIT STREAM (BSC):

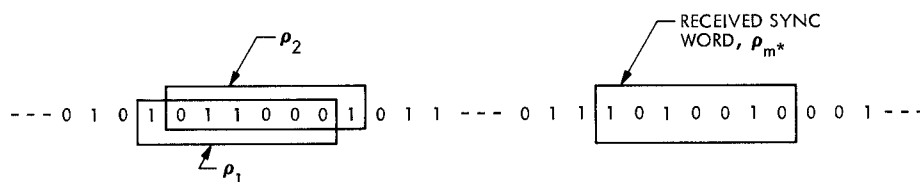


Fig. 1. Frame synchronization formats for uncoded transmission over a binary symmetric channel

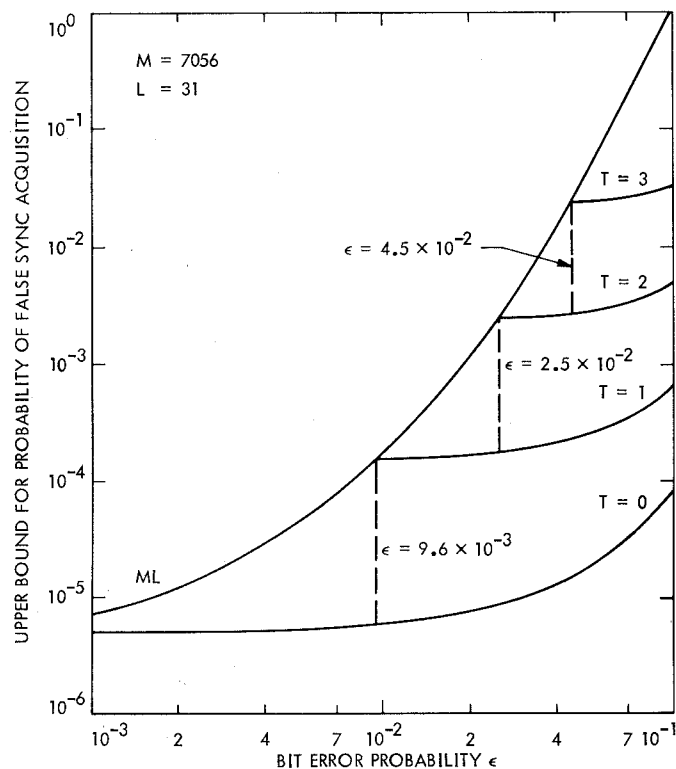


Fig. 2. Performance comparison of threshold and maximum likelihood frame synchronization techniques

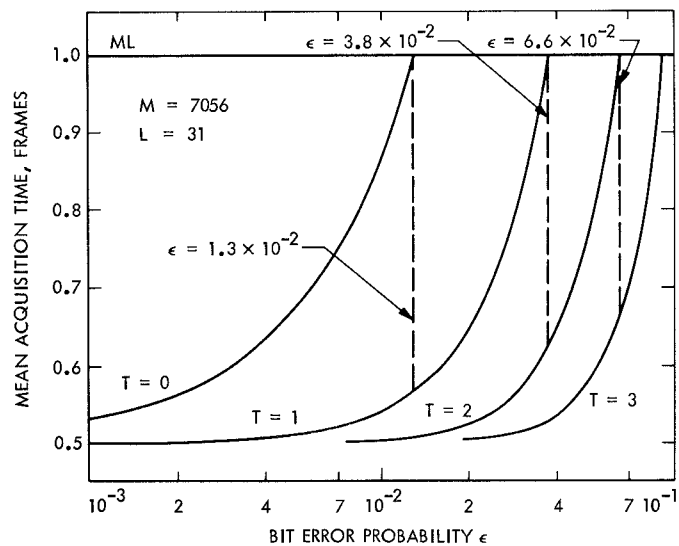


Fig. 3. Comparison of mean acquisition times for threshold and maximum likelihood frame synchronization techniques